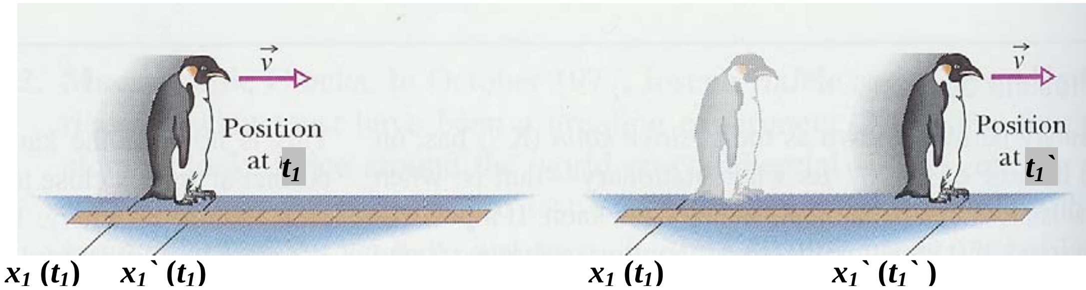
Length contraction
and
Time dilation

e-content for B.Sc Physics (Honours)
B.Sc Part-I
Paper-I

Dr. Ayan Mukherjee,
Assistant Professor,
Department of Physics,
Ram Ratan Singh College, Mokama.
Patliputra University, Patna

Length Contraction



If you want to measure the length of a penguin while it is moving, you must mark the positions of its front and back *simultaneously* (in your reference frame), as in (a), rather than *at different times*, as in (b).

Applying the Lorentz transformations to our two distances, we obtain;

$x_2 = \gamma\ (x_1 - vt_1)$ and $x_2` = \gamma\ (x_1` - vt_1)$

Subtracting, we obtain;

$$(x_2` - x_2) = \gamma\ (x_1` - x_1)$$

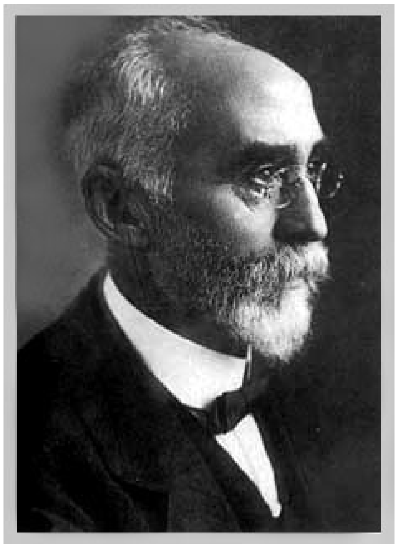
Note that $(x_2` - x_2)$ is the length as measured in S_2 . Since the object is at rest with respect to S_2, let's call this length L_o. This gives us

$$L = L_0\sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma}$$

Because the Lorentz factor γ is always greater than unity, then L is always less than L_o.

I.e. *The relative motion causes a length contraction.*

Because γ increases with speed v, the length contraction also increases with v.



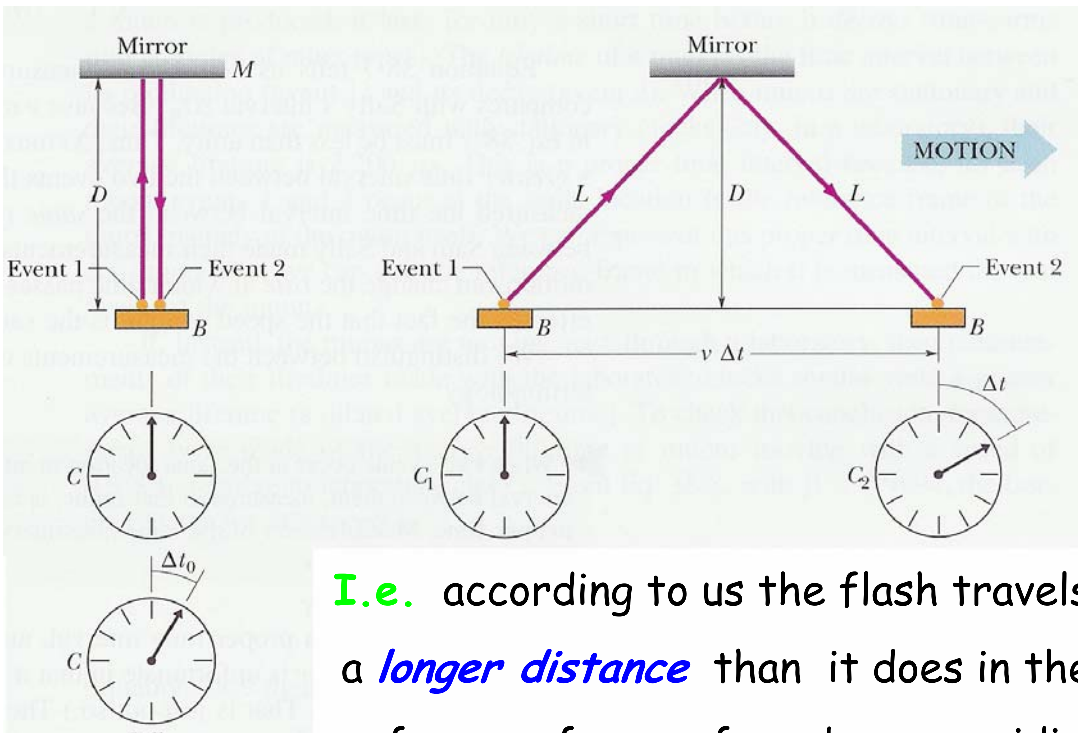
Remember: that $y_2 = y_1$ and $z_2 = z_1$.

Therefore, any lengths measured **perpendicular** to the direction of the motion **will not be changed** by the motion.

Length contraction occurs only along the direction of the relative motion.

Time Dilation

Suppose we travel inside a spaceship and watch a light clock. We will see the path of the light in simple **up-and-down motion**. If, instead, we stand at some relative rest position and observe the spaceship passing us by $0.5c$. Because the light flash keeps up with the horizontally moving light clock, we will see the flash following **a diagonal path.**



I.e. according to us the flash travels a ***longer distance*** than it does in the reference frame of an observer riding with the ship. Since the ***speed of light is the same*** in all reference frames (Einstein's second postulate), the flash must travel for a ***longer time*** between the mirrors in our frame than in the reference frame of an observer on board.

$$\Delta t_0 = \frac{2D}{c}$$

$$\Delta t = \frac{2L}{c}$$

$$L = \sqrt{\left(\tfrac{1}{2} v\Delta t\right)^2 + D^2}$$

$$\Rightarrow \quad \boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_0}$$

This stretching out of time is called **time dilation**.

Some numerical values:

* Assume that $v = 0.5c$,then $\gamma = 1.15$, so $T = 1.15\ T_0$ .This means that if we viewed a clock on a spaceship traveling at half the speed of light, we would see the second hand take 1.15 minutes to make a revolution, whereas if the spaceship were at rest, we would see it take 1 minute.

* If the spaceship passes us at 87% the speed of light, $\gamma = 2$; and $T = 2\ T_0$. We would measure time events on the spaceship taking **twice the usual intervals**. **i.e.** the hands of a clock on the ship would turn only half as fast as those on our own clock.

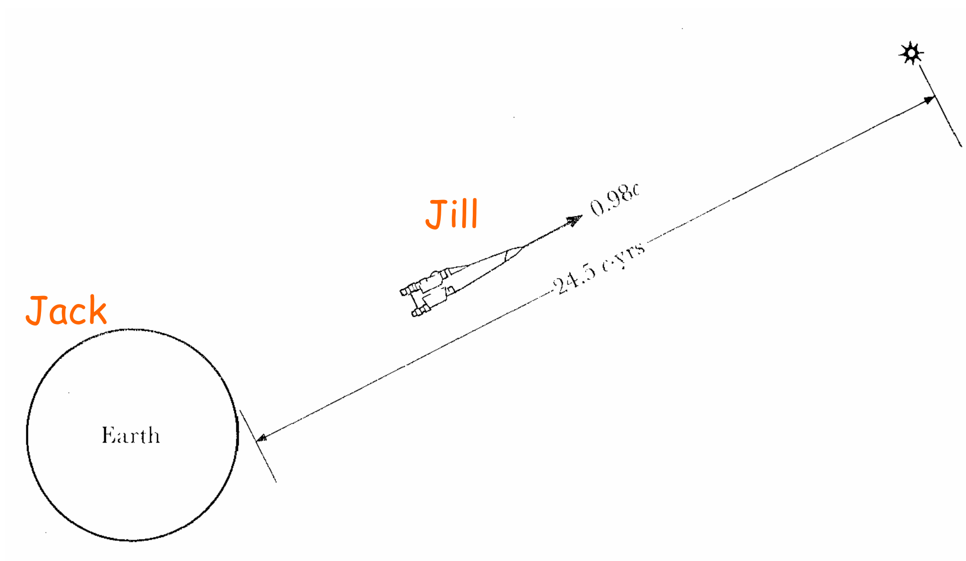
* If it were possible to make a **clock fly** by us at **the speed of light**, **the clock would not appear to be running at all**. We would measure the interval between ticks to be infinite.

* ***Time dilation*** has been confirmed in the laboratory countless times with particle accelerators. The lifetimes of fast-moving radioactive particles increase as the speed goes up, and the amount of increase is just what Einstein's equation predicts.

Example 4.6: (The Twin Paradox)

Jack and Jill are 25-year-old twins. Jack must stay on earth, but the astronaut Jill travels at **o. 98c** to a star **24.5** light years away and returns immediately. Ignoring the end-point acceleration times, find the twins' ages when she returns.

(One light year = 1 *C* . yr, the distance light travels in one year.)



Solution:

From Jack earth-bound frame of reference;

Jill travels a total distance of 49 light years (out and back) at 0.98c. Thus; the total time of her journey as *Jack* measure it is;

T_{Jack}= 49 c . yrs / 0.98c. = 50 years

Therefore 50 years of earth time have passed, so Jack is (25 + 50) years = 75 years old. However, this 50 years is dilated time for Jill's frame of reference.

Since γ = **5** for **v** = **0.98c,**

T_{Jill} = 50 years / 5 = 10 years

Jill therefore is (25 + 10) years = 35 years old. She is 40 years younger than her brother.

Question??

Since the choice of frame of reference is relative, **why don't we place Jill in S1?** She then sees the earth move away and return, and therefore it is Jack who has travelled out and back at 0.98c. He should be the one who is 40 years younger.

Since they both can't be 40 years younger, this apparent contradiction is called *the twin paradox*.

Answer:

❶ Recall that we are dealing with **the special theory of relativity, which refers to inertial reference frames.** In the twin paradox, the earth is an approximately inertial reference frame, but Jill's spaceship isn't. The choice of frames of reference is relative in the special theory of relativity *only if the frames of reference are all inertial*. Therefore an attempt to use the special theory in a no inertial frame of reference causes incorrect results. So, **Jack does age more rapidly than Jill.**

❷ Experiments (such as the clocks in jetliners) confirm this prediction.

❸ **Length contraction** can be used, as well, to solve this problem:

According to Jill spaceship frame of reference;

Jill travels (out and back)a total distance of:

$L_{Jill} = L_{Jack} / \gamma = 49$ c . yrs /5 = 9.8 c. yrs

Since she travels at 0.98c. Thus; the total time of her journey as *she* measure it is;

$T_{Jill} = 9.8$ c . yrs / 0.98c. = 10 years

Which confirm the previous prediction.